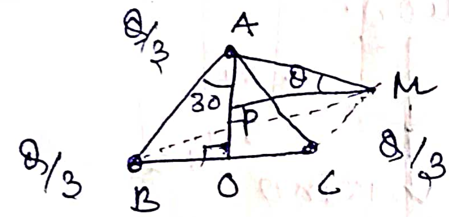
Problem:

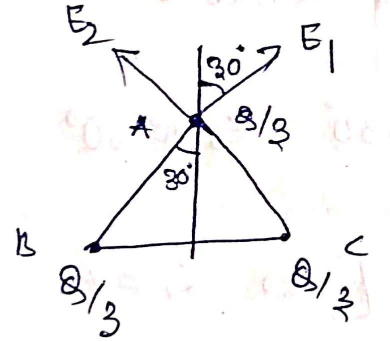
Three identical small spheres of mass m are suspended from a common point by threads of negligible masses. and equal length l. A charge Q is divided equally among the spheres, and they come to equilibrium at the corners of a horizontal equilateral triangle whose sides are d. Show that

$$Q^2 = 12\pi\epsilon_0 m g d^3 \left[l^2 - \frac{d^2}{3}\right]^{-1/2}$$

where g = acceleration due to gravity

<u>solution</u>

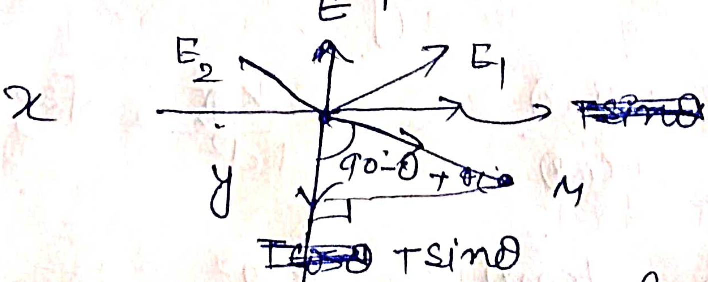




Let △ABC is equilateral formed by three charges.

AM is thread by which charge Q/3 is suspended at A.

- T be tension in thread AM on dissolving component of tension of thread at point A



E_1 is electric force on A due to B

E_2 is electric force on A due to C

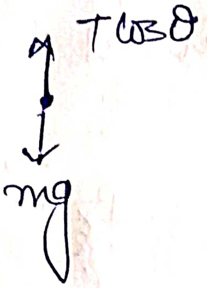
E is resultant of E_1 and E_2

$T \cos\theta \Rightarrow E$

On equating the force in x and y direction

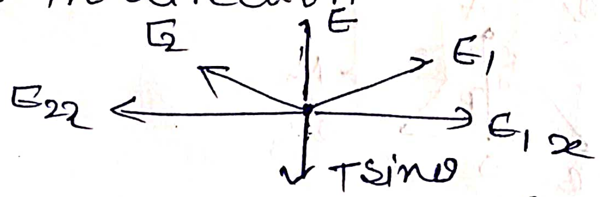
we get $T\cos\theta = mg$ — (1)

$T\sin\theta = 2 \times \frac{Kq^2}{9d^2} \cos 30^\circ$ — (2)



$E_1 = E_2 = \frac{Kq^2}{9d^2}$ — (3)

x component of E_1 and E_2 will cancel each other as both are equal in magnitude and opposite in direction.



$\therefore E_x = 0$

only y component will remain

so $E_y = E_1 \cos 30^\circ + E_2 \cos 30^\circ$ — (4)

$|\vec{E}| = 2E_1 \cos 30^\circ$ [as $E_1 = E_2$] — (5)

$\therefore$ on dividing equation (1) and equation (2)

$\Rightarrow \frac{T\sin\theta}{T\cos\theta} = \frac{2Kq^2}{9d^2} \times \sqrt{3}/2 \times \frac{1}{mg}$

$\Rightarrow \tan\theta = \frac{Kq^2}{9d^2} \frac{\sqrt{3}}{mg}$ — (6)

Now we find $\tan\theta$
From ΔAMP — (7)

$$\tan\theta = \frac{AP}{PM}$$

$AP = \frac{2}{3} AO$ [As centroid divides Median in ratio 2:1]

From ΔABO
we say $AO = \sqrt{AB^2 - (BO)^2}$
As ABO is right angle triangle
AO will be dividing BC in two equal parts
[property of equilateral triangle]

$\therefore AO = \sqrt{d^2 - \left(\frac{d}{2}\right)^2}$ [$AB = d$, $BO = d/2$]
$BO = BC/2$; $BC = d$

$$AO = \frac{\sqrt{3}\, d}{2}$$

$$AP = \frac{2}{3} \times \frac{\sqrt{3}}{2} d = \frac{d}{\sqrt{3}}$$

$AM = l$ (Given)

ΔAMP is right angle Δ

$\therefore PM^2 = AM^2 - AP^2$

$$PM = \left(l^2 - \left(\frac{d}{\sqrt{3}}\right)^2\right)^{1/2}$$

$\Rightarrow$

$$PM = \left(l^2 - d^2/3\right)^{1/2}$$

$$\therefore \tan\theta = \frac{d}{\sqrt{3}} \times \frac{1}{(l^2 - d^2/3)^{1/2}}$$ — (8)

From equation ⑥ and equation ⑧

$$\Rightarrow \tan\theta = \frac{KQ^2}{9d^2}\frac{\sqrt{3}}{mg} \quad -⑥$$

$$\Rightarrow \tan\theta = d/\sqrt{3} \times \frac{1}{(l^2-d^2/3)^{1/2}}$$

$$\Rightarrow \frac{d}{\sqrt{3}} \times \frac{1}{(l^2-d^2/3)^{1/2}} = \frac{KQ^2}{9d^2}\frac{\sqrt{3}}{mg}$$

$$\Rightarrow Q^2 = \frac{9d^3 mg \,(l^2-d^2/3)^{-1/2}}{3K}$$

$$\Rightarrow Q^2 = \frac{3d^3 mg}{\frac{1}{4\pi\epsilon_0}} (l^2-d^2/3)^{-1/2}$$

$$\Rightarrow Q^2 = 12\pi\epsilon_0 d^3 mg \,(l^2-d^2/3)^{-1/2}$$

Question for practices

Q.1 The magnitude of the electric field at a distance of 1m from a point charge is 1×10^5 N/C. What is the magnitude of electric field at a distance of 2m & 3m?

Q.2. Given that an electric field of 10 N/C has a northward direction and another electric field of 10 N/C has an eastward direction, what are the magnitude and direction of the superposition of these two electric fields?

Q.3. Four equal positive charges are located at the corners of a square. What is the magnitude of electric field at the center of the square?

Q.4. Find the electric field a distance z above the midpoint between two equal charges, q, a distance d apart as given in figure below.